An Options Approach to Commercial Mortgages and CMBS Valuation and Risk Analysis

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INTRODUCTION

Investment in commercial mortgages and commercial mortgage-backed securities (CMBS) has received increased attention from mainstream fixed-income investors. Yet, much of the quantitative technology that has been developed for analyzing relative value in such areas as residential mortgage-backed securities (MBS) and corporate bonds has not been applied to commercial mortgages. Investors in agency mortgage-backed securities and callable corporate bonds, for example, have used option pricing to determine fair value for securities whose cash flows are uncertain due to the possibility of an early call. Commercial mortgages typically have greater call protection than residential MBS and corporate bonds, although they still have some callability. Most commercial mortgages have lock-out periods followed by a period during which a penalty is applied to premature principal payments, followed in turn by a free period. The technology applied to other fixed-income instruments to value these features could be used for assessing risk and relative value in commercial mortgages and CMBS.

In addition to the risk of early principal payment, commercial mortgages, like corporate bonds, are subject to the risk of losses in a foreclosure following a default. Pricing methodology described in academic journals has been applied to this risk for corporate bonds, but for a variety of reasons it has not proven to be practical.[1] The analysis and valuation of this risk for commercial mortgages using similar quantitative analysis, while discussed in the academic world, has not been applied in the market for commercial mortgages.[2]

The need for improved tools has increased with the introduction of securitization, where the most popular method of credit enhancement is the senior-subordinated structure. Whole loans, which are typically of BB quality, are aggregated and their cash flows are then allocated to create securities with credit ratings from AAA down to B and unrated. As a result, the risk of loss due to default is leveraged up in the junior classes and leveraged down in the senior classes. The analogy in the residential MBS area is the creation of planned amortization class (PAC) bonds and support tranches where the PAC bond has leveraged down prepayment risk and the support bond has leveraged up prepayment risk. Moreover, the senior subordinated structure requires that recoveries from foreclosures first be used to pay senior bondholders. From the perspective of these bondholders a prepayment event has occurred, even though the unscheduled cash flow came about due to a credit event. Nevertheless, it must be considered in the valuation and risk of these AAA rated securities.

When an investor looks at a CMBS deal it would be useful to know whether or not the AAA class at 90 basis points over Treasuries is a better value than the B class at a +600 basis point spread. How should one compare the risk of a bullet loan with one that amortizes?

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[1]The complexity of the capital structure of a corporation and the possibility of a leveraged buyout make the application of the option approach less practical for corporate bonds. For more details on the option approach to pricing default risk in high yield bonds, see Richard Bookstabber and David P. Jacob, "Controlling Interest Rate Risk," Chapter 8 in *The Composite Hedge: Controlling the Credit Risk of High-Yield Bonds* (New York: John Wiley & Sons, 1986).

[2]There has been some work published in this area. See Chapter 12 and Patrick *J.* Corcoran "Commercial Mortgages: Measuring Risk and Return," *Journal of Portfolio Management* (Fall 1989); Sheridan Titman and Walter Torous, "Valuing Commercial Mortgages: An Empirical Investigation of the Contingent Claims Approach to Pricing Risky Debt," *Journal of Finance* (June 1989); and, Paul D. Childs, Steven H. Ott, and Timothy S. Riddiough, "The Pricing of Multi-Class Commercial Mortgage-Backed Securities," Working Paper (December 1994).

How does the risk of default affect the value of a security trading at a premium? What is the fair value of an interest-only strip when the loans underlying the deal have percentage penalties versus yield maintenance, versus lockout. This chapter describes a two-factor contingent-claims theoretic framework and applies option pricing methodology to commercial mortgages to answer these questions.

In the next section we outline the elements of valuation and the basic analytic approach to pricing commercial mortgages. Following this, we apply the approach to commercial whole loans and show the effects of each factor on the value of the mortgage. Next, a multi-class senior-subordinated deal is evaluated. We then use the model to look at the relative risk of different securities. Finally, we draw some conclusions, discuss practical issues relating to the model, and propose some future applications. In the appendix we show some of the mathematics behind the model.

THE ELEMENTS OF VALUATION

The value of all real estate securities is contingent upon the value of the underly-ing real estate asset since they each have a claim on this asset. For example, the equity holder has a residual claim on the income stream after the debt holder is paid. If the income from the real estate asset is insufficient to meet the debt obli-gation and a default results, the debt holder has a claim on the real estate. Usually, the equity holder defaults only when the value of the real estate is less than the value of the loan and when income is insufficient to pay debt service. The debt holder, in this case, will receive the smaller of the debt payment or the value of the real estate and the equity holder receives nothing.

The analytic approach we use is to view the owner (lender/investor) of a commercial mortgage as having a long position in a credit risk-free, non-callable mortgage, a short call option, and a short put option. The commercial mortgage investor/lender (debt holder) has written an option to the borrower (equity holder) to call (prepay) the debt, and an option to put (default) the real estate to the debt holder. That is,

Commercial mortgage = (Default-free and non-callable mortgage)
- (Call option) - (Put option)
PREPAYMENT DEFAULT

As compensation for writing these options the debt holder receives a spread over the yield on Treasury bonds usually in the form of a higher coupon. Therefore, in order to value the commercial mortgage, one can value the risk-free cash flows and the associated call (prepayment) and put (default) options. To properly value the options, the default and prepayment options need to be analyzed simultaneously since as we will show they are interrelated.

To value the options we need to define what circumstances would cause the property owner to exercise his options.

Prepayment option — triggering conditions: Prepayment is triggered for two reasons:

a. economic benefit from refinancing which occurs if
    1. the general level of interest rates drop.

or
    2. the property value increases, thus allowing the borrower to refinance at a tighter spread to Treasuries.

or
b. Owner wants to sell property and the mortgage is not assumable.

For condition *a* to be viable, net operating income (NOI) must be sufficiently greater than the scheduled payments required under the new rate, since otherwise the borrower would not qualify for the loan. If the borrower does qualify, he will refinance so long as the present value of the future promised payments minus the value of the options (fair market value of the debt including its embedded options) is greater than the face value of the remaining debt plus refinancing costs such as prepayment penalties. Thus, as interest rates drop (for newly originated fixed-rate mortgages) and as the quality of the property improves the likelihood of refinancing increases since under these circumstances the market value of the debt increases.

In addition, property owners sometimes want to realize the return on their properties particularly as the tax benefits of ownership decline through time. If the mortgage is assumable or a substitution of collateral is permitted, the owner could sell the property with the loan remaining intact. Otherwise the owner would have to prepay the mortgage. Another situation that could occur that would lead to prepayment even in a rising rate environment is if the property appreciates in value, and the owner desires to re-leverage the property. If the mortgage note prohibits additional financing (this almost always the case for CMBS), then the borrower must first repay his loan.

Empirical evidence on commercial mortgage prepayments by Abraham and Theobald reported in Chapter 3 of the first edition of this book suggests that when it is economic for commercial property owners to prepay, they do so at an even faster rate then owners of residential properties. Moreover, turnover rates in property ownership indicate that even if refinancing is uneconomic property owners sell their properties to realize profits. For example, Abraham and Theobald found that the cumulative prepayment rate for low coupon mortgages that were outstanding for 10 years was 82.4%.[3]

Default option-triggering conditions: For the property owner to exercise his default option there are two necessary conditions.

i) Net operating income is less than the current period's scheduled mortgage payment[4]
and
ii) The market value of the property is less than the market value of the debt[5]

For a non-callable mortgage, default will never be necessary for a rational borrower if the NOI is enough to cover debt payment. Default starts to occur when the NOI is insufficient to meet the debt service. When that happens and the property value is also less than the value of the debt, the default option would be exercised. Both conditions are necessary because if the property value is greater than the value of the debt, but the NOI is insufficient to pay the debt service, the property owner would attempt to sell the property and payoff the debt rather than go through foreclosure.

Default as a method prepayment — triggering conditions: Sometimes the property owner may try to use a default as a method of prepaying so as to avoid the prepayment penalty and/or lock-out feature.[6] In this case, the triggering conditions for default are more complicated. The conditions would be triggered to default as follows:

i) The NOI has to be greater than the payments that would be required at the time if the loan were to be refinanced.
and
ii) the present value of the future promised payments minus the value of the options (fair market value of the debt including its embedded options) is greater than the face value of the remaining debt plus foreclosure expenses.

If these two conditions hold the borrower can default, go through foreclosure, pay off the face value of the debt with the proceeds, and then refinance. This situation can arise when interest rates drop and property value and NOI increase, but the loan is either locked-out or there is a stiff prepayment penalty. In this case if the foreclosure expenses are not too onerous, the borrower has an incentive to default. It is unclear, however, how the courts would treat this situation. It is possible that the bankruptcy judge would force the borrower to compensate the lender.

[3]This is one reason why interest strips from CMBS deals that have lockout provisions as opposed to simple yield maintenance are far less risky and should trade at tighter spreads.
[4]Net cash flow might be more appropriate, but here we use NOI for simplicity.
[5]The market value of the debt is the present value of the future promised payments plus the current payment that is due minus the value of the options.
[6]Experts in bankruptcy law feel that in a true default, prepayment penalties could be construed by the judge as usury and therefore disallowed.

The Combined Default and Prepayment

Since the call and put options are embedded in the mortgage debt, the call option and the put option cannot actually be separated. The incentive to prepay as we have discussed is linked not just to the general level of interest rates, but to the ever changing level of operating income of the property and the resulting available refinancing spread. Thus, the value of the prepayment option is related to factors that affect the value of the default option. Similarly the incentive to default is related to the level of interest rates which in turn affects the value of the prepayment option. Moreover, borrowers who either prepay or default terminate the contract of the mortgage. This results in the termination of both options. Our triggering conditions, thus, do not work independently, but need to be evaluated simultaneously.

To visualize the triggering process for the prepayment option, look at Exhibit 1. The horizontal axis measures time. The vertical axis tracks interest rates which in turn determines the present value of the promised payments. As time passes, interest rates can move up or down. As interest rates drop, the market value of the debt increases above the face value making it economically worthwhile for the borrower to refinance.

A lock-out or penalty reduces the value of the call option since it lessens the likelihood of the option being exercised. In general, the longer the term to maturity and the more volatile the interest rate, the more valuable the prepayment option since the likelihood of exercise increases.

In order to visualize the triggering process for a loan default, we make use of the metaphor of a drunk person walking along the edge of a cliff trying to go from point A. to point B. The closer he is to the edge when he begins his walk, the more erratic his walk, and the longer the distance from point A to point B, the more likely he will fall off the cliff before reaching point B. Similarly, in the case of an income generating property, the more volatile the NOI, the greater the initial loan-to-value (LTV), the lower the debt service coverage ratio (DSCR), and the longer the maturity of the debt, the higher probability of default prior to maturity. In Exhibit 2 the horizontal axis measures time to maturity. The vertical axis measures the level of NOl and LTV. As time passes NOI and LTV can move up or down. If NOI/LTV moves down/up sufficiently, the property owner will default and hand the keys of the property to the lender.

Exhibit 1: Prepayment Option of a Commercial Mortgage

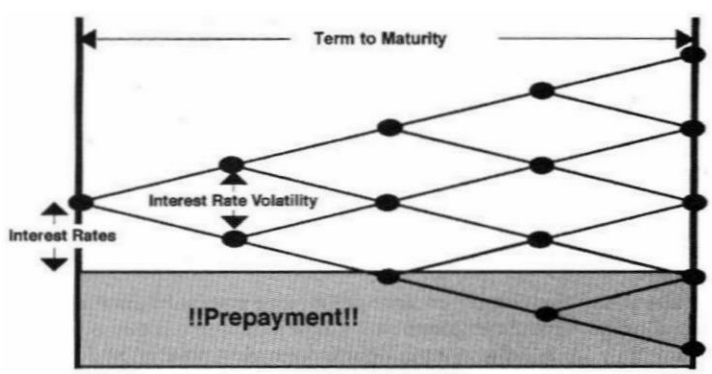
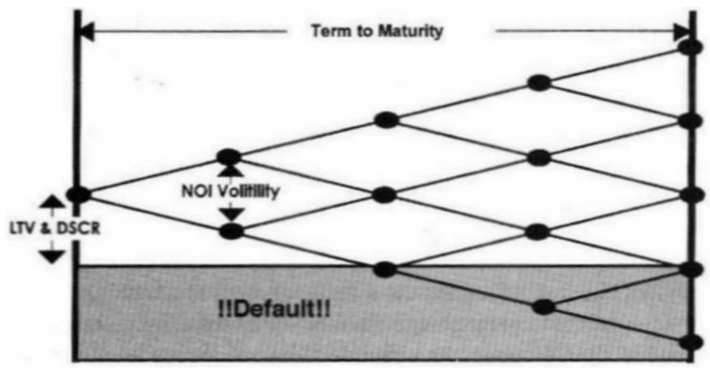


Exhibit 2: Default Option of a Commercial Mortgage



Determinants of Option Values

Now that we have defined the conditions that lead to the exercise of the options, we need to identify the determinants of the options' values. The value of the embedded options depends upon many factors. The direct determinants are

1. Current balance of mortgage
2. Term to maturity of mortgage
3. Mortgage payments including interest and principal, and the amortization schedule
4. Prepayment terms and penalties
5. Net operating income from the collateral property
6. Volatility of net operating income
7. Terms of default and foreclosure costs
8. Interest rates
9. Volatility of interest rates
10. Correlation between interest rates and net operating income

The first four items specify the information necessary to calculate the promised cash flows of the underlying mortgage. This information in conjunction with current and the potential future interest rates are necessary for calculating the value of the prepayment option. Items 5, 6, and 7 which relate to the property are essential for valuing the default option. The last three items are critical for valuing all assets, including the mortgage, the real estate, and the options.

The Valuation Process

The process of interest rates and net operating income determine the entire valuation procedure of the commercial mortgage and its embedded options. In our framework, we assume that interest rates and NOI are the two underlying building blocks. The property value which is the present value of all future NOI's can be calculated from interest rates and NOI. To solve for the option values, a two-dimensional binomial tree or pyramid is constructed (by combining Exhibit 1 and Exhibit 2) based on the assumptions of the process and volatility governing future NOI and interest rates.[7]

For every path of interest rates there is a whole set of possible paths of NOI. The tree will specify the future cash flows

[7]When a multi-class commercial mortgage-backed security is evaluated, the path-independent condition for the remaining balances of the bond classes does not necessarily hold. Fortunately, so long as the underlying loans satisfy the path-independent condition, Monte Carlo simulations which randomly select a finite number of paths from a virtually infinite number of paths can be utilized to calculate the option values. Since a huge path selection process is involved, variance reduction techniques turn out to be very important to improve the sampling method.

of the mortgage under the full range of interest rates and NOI scenarios. Once the pyramid is created, the value of the property can be calculated at each node of the pyramid.[8] Similarly other relevant variables such as LTV and DSCR can be calculated as well. At each node, the action taken by the borrower (prepayment, default, or scheduled payment) determines the cash flow that the debt holder receives. The option values and the fair value of the mortgage can then be calculated by discounting the cash flows backward in time through the pyramid. The values are equal to the expected discounted value of the cash flows through the pyramid. The theoretical or fair value of the commercial mortgage can be obtained by combining these terms.

Option-Adjusted Spread

Since the market value or market price of a financial security may differ from its fair value, the fixed-income market has developed the concept of option-adjusted spread (OAS).

OAS is a spread relative to the Treasury curve, quoted in basis points, which is used for measuring the relative value of securities with a series of uncertain cash flows. The OAS can be obtained by calibrating the theoretical present value to the current market price. The theoretical present value takes all possible cash flow streams discounted by the corresponding discount rates and weighted by assumed probabilities. The OAS, thus, is a constant spread added to the risk-free interest rate and is used as the discount rate for the corresponding cash flows. The procedure involves solving for the spread which equates the price obtained via discounting the cash flows to the market price.

The larger or more positive the OAS, the cheaper the security is relative to its theoretical value. The OAS can be thought of as the risk premium which the investor would earn if he repurchased or hedged, at fair value, the options that he has implicitly shorted by owning the security. The concept of OAS was originally introduced to analyze relative value in residential mortgage-backed securities and callable corporate bonds, where the borrower's prepayment option substantially negatively impacts the value of these securities. If the OAS is positive/negative, then the investor is receiving more/less than he should have for shorting the embedded options.

Parameter Estimation and Practical Considerations

Like all option models, a number of parameters need to be estimated and assumptions need to be made regarding the process governing the random variables. In our case, we need to have an estimate for the volatility of NOl for the property, the volatility of interest rates, and the correlation between these. If the loan or a security is backed by a number of properties, then we also need the correlation matrix of NOl of all the properties.

Regarding interest rates there is a voluminous body of literature which addresses the interest rate process necessary to properly price fixed-income income options and to satisfy the arbitrage-free condition for the term structure of interest rates.[9] In this chapter, we use the Black, Derman, and Toy model and its binomial free to calibrate the interest rate lattice to the initial yield curve.

To empirically estimate the NOl volatility, we first used the Russell/NCREIF Property Index (RNPI) despite all of its drawbacks.[10] The RNPI index is the most widely quoted index for real estate property performance. It provides data

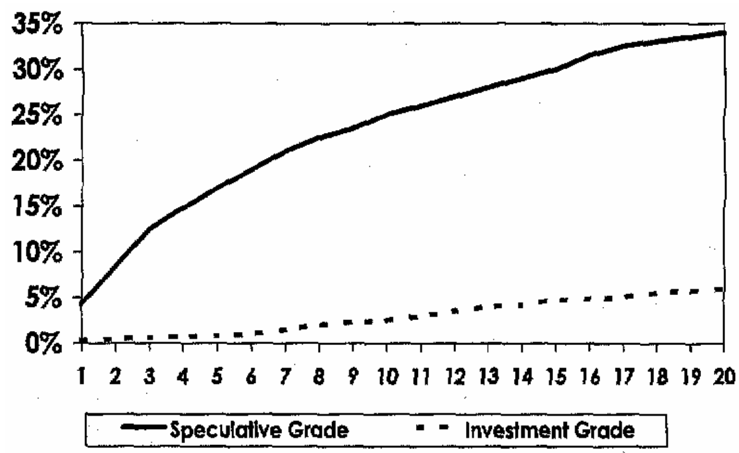
[8]In our model we specify a term which indicates the growth rate, if any, in the NOI. We define the property value to be equal to NOI/(R - R x G) where R equals prevailing interest rates, NOl is net operating income, and R x G is the growth rate in NOI. R - R x G equals the traditional cap rate.

[9]Ho and Lee used a binomial tree to create an arbitrage-free interest rate model. See Thomas S.Y. Ho and S.B. Lee, "Term Structure Movements and Pricing of Interest Rate Contingent Claims," *Journal of Finance* (December 1986). Black, Derman, and Toy also constructed a binomial tree model and, furthermore, allowed various volatilities for the entire term structure. See Fischer Black, Emanuel Derman, and William Toy, "A One-Factor Model of Interest Rates and its Application to Treasury Bond Options," *Financial Analysts Journal* (January-February 1990). Hull and White created a trinomial tree and provided a closed form solution for arbitrage-free model in continuous time. See John Hull and Alan White, "One-Factor Interest-Rate Models and the valuation of Interest-Rate Derivative Securities," *Journal of Financial and Quantitative Analysis,* (June 1993). Both Jamshidian and Chan assumed a variety of interest rate process to calibrate to the term structure. See Farshid Jamshidian, "Forward Induction and Construction of Yield Curve Diffusion Models, "*Merrill Lynch Research* (March 1991); and, Y.K. Chan, "Term Structure as a Second Order Dynamical System, and Pricing of Derivative Securities," *Bear Stearns Research,* 1992.

[10]The Russell/NCREIF property index is an appraisal-based index of property returns and values. This index represents data collected from the Voting Members of the National Council of Real Estate Investment Fiduciaries. Many researchers feel that the appraisal process causes the data to significantly understate the price volatility. The volatility of the income component, on the other hand, might be representative for a large pool of assets.

such as net operating income and appraisal value of commercial buildings by region and by property type on quarterly basis. We also used the RNPI series to estimate the correlation between interest rates and net operating income. The 6% volatility of NOI that was estimated from the large and diversified pool of properties that underlie the RNPI series greatly understates the true NOI volatility of individual properties. Based on our own data we expect the volatility of NOI to range between 9% and 16%. There are differences by property type. As we would expect multi-family properties have lower NOT volatilities and hotels tend to have higher volatilities. In practice one needs to estimate the volatility for the property in question.[11]

Exhibit 3:Average Cumulative Default Rates for Corporate Bonds



Source: Moody's Investors Service.

One way to calibrate the NOI volatility assumption is to see what pattern and level of defaults is produced using the parameter assumptions. In Exhibit 3 we show the actual average cumulative default rates for investment grade and speculative grade corporate bonds as a function of time as computed by Moody's Investors Service for the years 1970-1994. One can see that for the high quality debt the marginal default rate starts out low and increases over time. On the other hand, the marginal default rate (as measured by the slope of the curve) for the speculative grade debt declines over time. The reason for this is that lower quality debt that survives the early years has a decreasing probability of defaulting. Whereas, high quality debt has an increasing (or at least non-decreasing) chance of defaulting as time passes.

In Exhibit 4 we show the marginal default rates for high quality (low LTV) and low quality (high LTV) loans that are implied by our model using an assumption of 16% volatility of NOI. In Exhibit 5 we show the implied cumulative default rates. The pattern is very consistent with the Moody's data and demonstrates the ability of the model to differentiate the default pattern associated with different quality loans. Moreover, the level of defaults, while higher than what Moody's found for corporate bonds, in our view represents a reasonable level of defaults.[12]

Aside from parameter estimation, there are some practical considerations when implementing the model that differ from the theory. The theory assumes the following conditions: (1) liquidity, (2) symmetry of market information, (3) optimal

[11]Specific features such as cross-collateralization, which would lower NOI volatility need to be modeled.

[12]The cumulative default level after 10 years implied the by the model for a 70% LTV loan with an initial DSCR of 1.5x using a 16% NOI volatility is about 26%. Moreover, the implied loss severity is about 19%, both of these statistics exceed the levels found with the historical data. We feel that our standard of 16% NOI volatility is reasonable.

exercise, and (4) refinancing ability.

Exhibit 4: Implied Marginal Default Rates for Commercial Mortgages

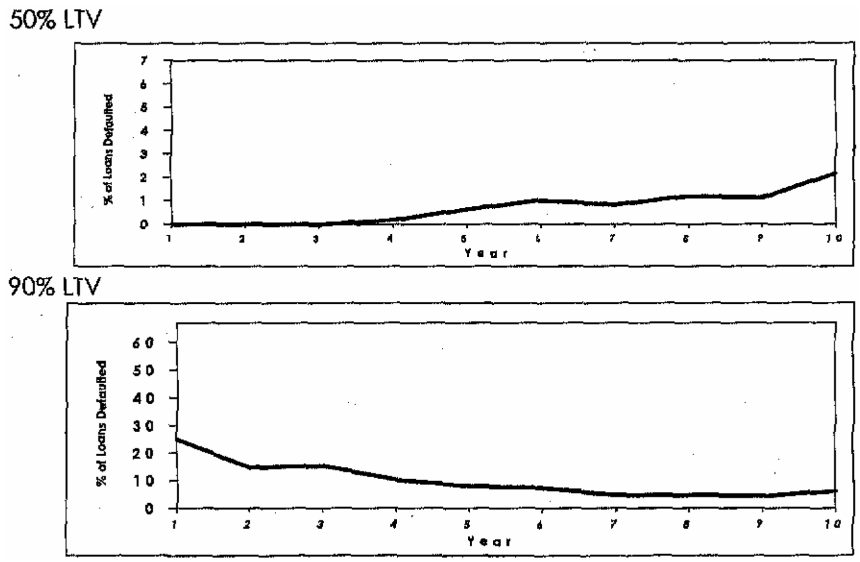
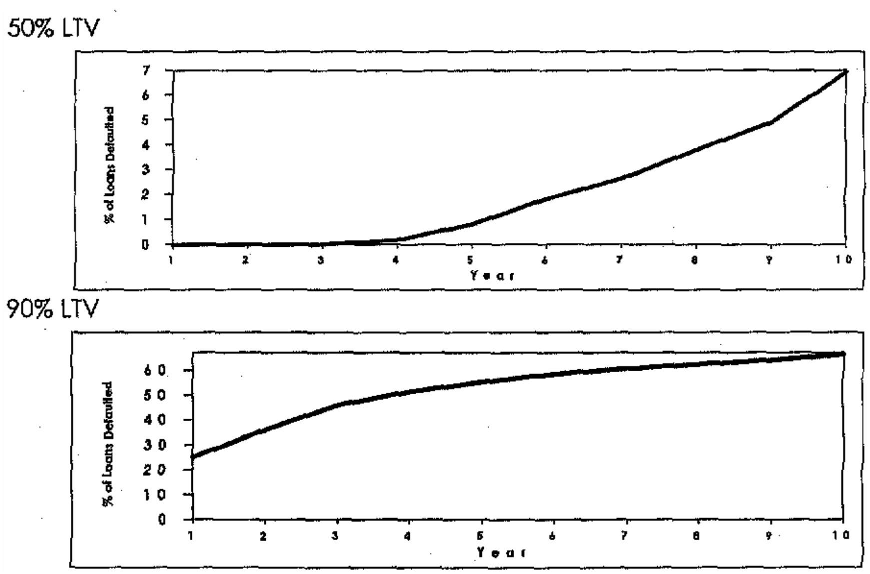


Exhibit 5: Implied Cumulative Default Rates for Commercial Mortgages



Liquidity refers to how easy it is to buy or sell real estate and commercial mortgages in the secondary market. The theoretical model assumes that transactions costs are low and that the markets are very liquid. Symmetry of market information refers to whether borrowers and investors have the same information. Optimal exercise refers to how efficiently the borrower exercises his options if arbitrage opportunities emerge. Refinancing ability refers to the fact that financing is based solely on the property consideration irrespective of the borrower. Even though many of these conditions do not hold in practice, the model provides value by incorporating in a single framework the primary factors driving the value of the mortgage. As a result it becomes useful as a relative value tool and for comparing risk.

COMPARATIVE ANALYSIS

In this section we use OAS as the measure to analyze the factors that affect the value of a commercial mortgage. In the next section, we apply the framework to a multiclass CMBS structure. In order to use the model to analyze the factors affecting the value of a commercial mortgage, we could either keep the spread constant (OAS), and see how the price varies as we change each of the factors. Or, we could keep the price constant and see how the factors affect the OAS as we change each of the factors. We adopt the latter approach since the market sets the price.

Since the OAS is considered compensation for shorting options, the OAS decreases for a fixed mortgage price as the default or prepayment risks increase. The degree of risk exposure depends upon the characteristics of the mortgage debt, the performance of underlying collateral as well as the economic factors such as interest rates. We analyze default risk with respect to four factors — DSCR, LTV, volatility of NOI, and mortgage maturity date. These factors are the usual measures of risk quoted in real estate markets for analyzing default risk.

In this section we use as an example, a commercial mortgage with a 10-year maturity (9.4 year weighted-average life), 7% coupon, initial DSCR of 1.30x, initial LTV of 70%, initial 10-year Treasury at 4.76%, and volatility of 11% and 16% for interest rates and NOI, respectively.[13] The initial spread of the loan is +226 bp over the 9.4-year interpolated Treasury. A correlation of 0.2 between interest rates and NOl is used. We assume that the loan follows a 30 year amortization schedule and pays a balloon payment at the end of year 10. *Initially, we assume that the mortgage is non-prepayable.*

Debt Service Coverage Ratio (DSCR)

The debt service coverage ratio (DSCR) is defined as the annual NOI divided by annual cost of debt service including principal payments. Normally, as DSCR gets larger, the probability of default and a resulting loss decreases. Thus, as DSCR increases the value of the default option decreases. As a result, for a fixed price the OAS increases as DSCR decreases. This can be seen in Exhibit 6 for a wide range of initial DSCR. Moreover, as DSCR gets very large, further increases do not result in a higher OAS.[14] For lower initial values of DSCR, the OAS is lower, indicating that the investor is really getting on average something less than +226 bp implied by his coupon. For example, at a DSCR of 2.50x the OAS is 188 bp, or equivalently the default option is worth +38 bp.

As one would expect the model shows that the OAS line for a bullet loan always lies below the line for the amortizing loan, since the amortizing loan has a built-in risk reduction mechanism. Even though both loans start with the same LTV, the LTV of the amortizing loan decreases through time, thus reducing its risk. The model computes the value of the amortization to be worth 30-40 bp.

Loan-To-Value Ratio (LTV)

The loan-to-value ratio (LIV) is defined as the ratio of loan amount to the value of the collateral property. This ratio is frequently used as a measure of leverage to assess the level of protection from default. A lower LTV loan is considered more credit worthy due to its better default protection. In terms of OAS, the OAS should increase as the initial LTV drops since the probability of a default and a loss decreases. This can be seen in Exhibit 7. When the LTV is sufficiently low,

[13]To reduce the number of calculations we assumed a semi-annual pay mortgage.
[14]Since we kept initial LTV constant at 70%, the OAS does not increase much above 195 bp. Thus, the probability of default does not go to zero. In reality LTV would likely go lower and the OAS would go to +226 bp. Similarly, as the DSCR gets lower, LTV should get lower and lead to lower OAS.

the loan is so default-protected that the OAS will start to level off. Lower initial LTVs do not lead to further increases in OAS. At this point the probability of default is near zero and the OAS equals the nominal spread of +226 bp.

Exhibit 6: OAS versus DSCR: Default Option

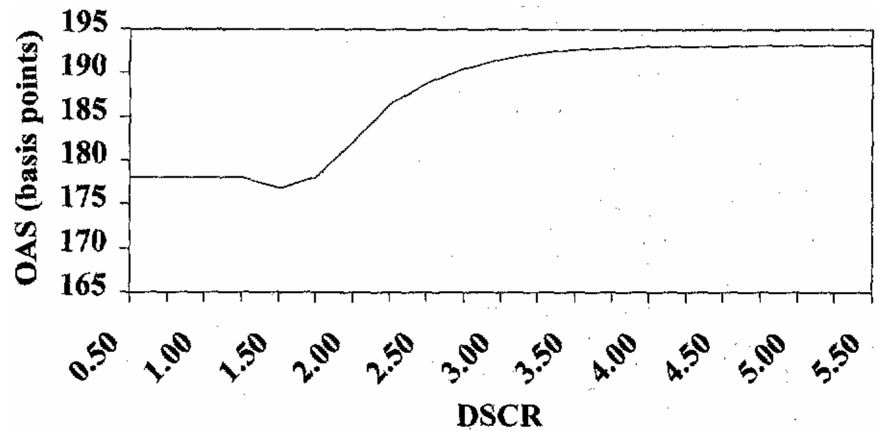
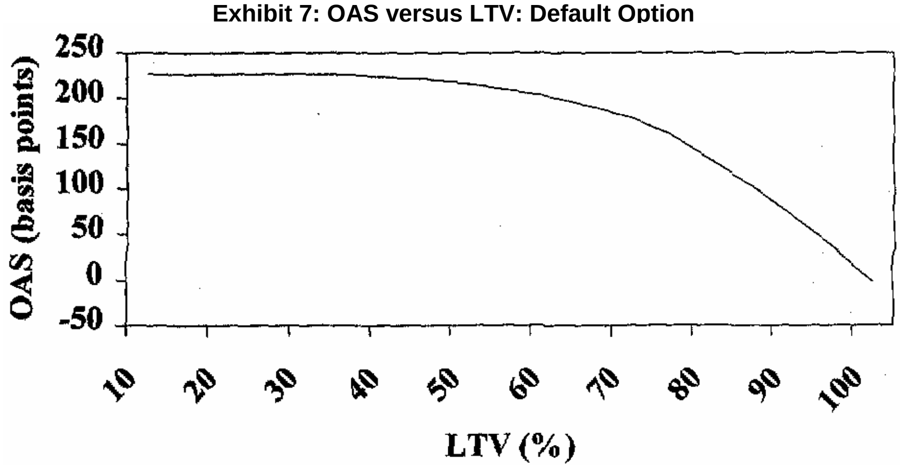


Exhibit 7: OAS versus LTV: Default Option



The impact on OAS due to change of LTV is more significant for bullet loans than amortizing loans.[15] This is because, as we stated earlier, the amortizing loan has a built-in LTV decreasing mechanism to automatically reduce default risk over time.

Volatility of Net Operating Income

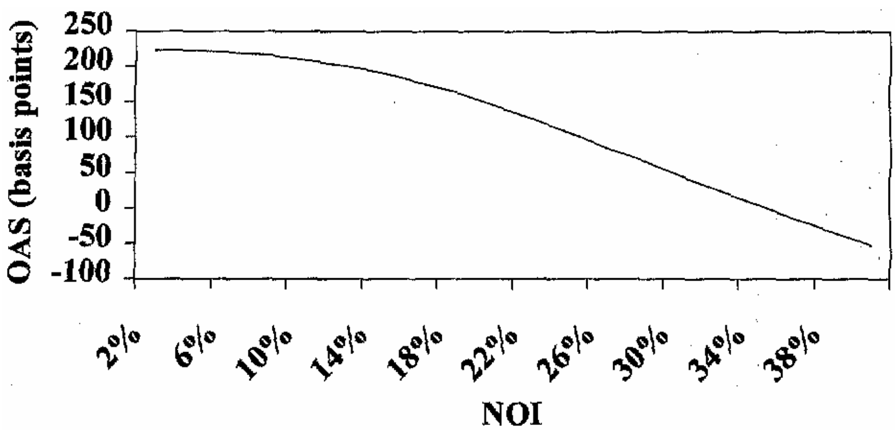
The more volatile a property's income stream, the greater the probability of default. From the model's perspective volatility of NOI affects value because with greater volatility there will be more paths under which income will be insuf*ficient* to pay the debt service and under which the value of the property declines below the value of the debt (see

[15]A bullet loan pays no principal until the maturity and then the entire principal amount is fully paid. The amortizing loan pays principal according to its amortization schedule until the maturity and then the remaining principal amount is fully paid.

Exhibit 2). Thus, given a mortgage price, the corresponding OAS decreases as the volatility increases. Exhibit 8 clearly indicates this result: the greater the volatility of NOl, the lower the OAS. It should be clear from the graph that at a 70% initial LTV, the volatility of NOI has a profound influence on the value of the loan.[16]

This parameter enables investors to estimate the required excess spread for different property types. For example, mortgages on hotels or other property types with relatively high operating margins should offer higher spreads than mortgages on property types such as multi-family which tends to have a more stable income stream, assuming the same initial LTV and NOl. In practice, loans and CMBS backed by hotel or office properties tend to have lower initial LTVs (in the case of CMBS greater subordination which translates into a lower effective LTV) and therefore the difference is not reflected in the spread.

Exhibit 8: OAS versus Volatility of NOI: Default Option



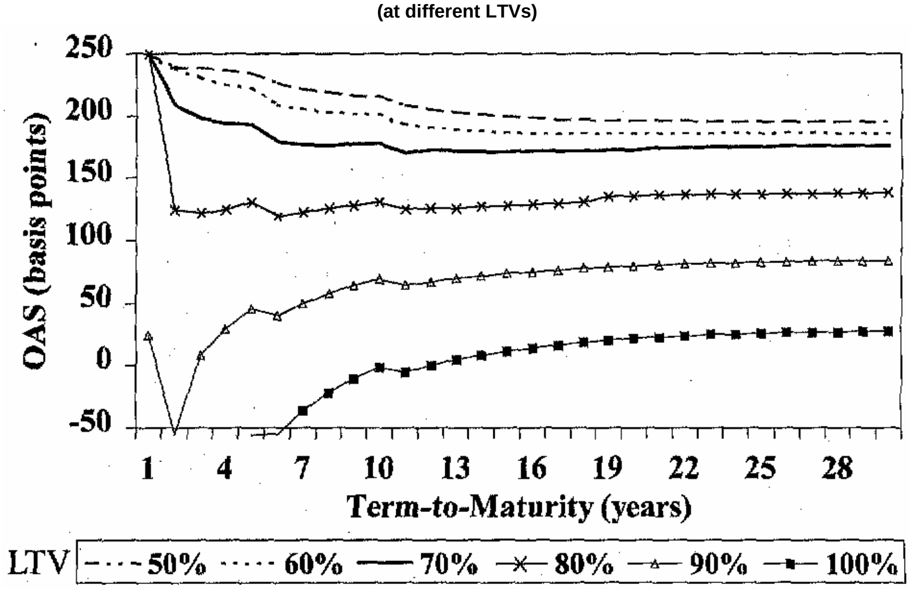
Term to Maturity

As previously shown in Exhibit 2, the longer the term to maturity, the higher default risk is. This is because there is more time for things to go bad. The cumulative probability of default increases as the term to maturity lengthens. As a result, the OAS should decrease as the term to maturity lengthens. Our model shows that OAS decreases as term to maturity increases, particularly for the higher quality loan (see Exhibit 9). On the other hand, when the loan starts out as a lower quality loan, increasing the maturity of the debt can actually lead to increasing OAS indicating increasing value. At first this seems strange, since the probability of default should increase with time. There are two reasons for the odd result. First, even though the cumulative probability of default increases, with a longer term to maturity the time to default is pushed further into the future.[17] Second, when a loan is really at risk of default, additional time gives the property owner some chance of getting the property back on track. An analogy can be made to a sporting event. Suppose a team is down by seven runs. It would far prefer it to be the first inning instead of the ninth, whereas the team that is ahead by seven runs would prefer that it be the bottom of the ninth inning.

Another interesting phenomenon is that even for the higher quality loan, after a certain point the OAS stops declining and even rises a bit until it levels off. This happens because even though the probability of default increases, on a present value basis, this added risk of default does not add much to the expected loss. The result is interesting in that it shows that longer term to maturity does not necessarily mean greater risk.

[16]Note that for sufficiently high LTV, as NOI volatility increases to a certain level, the OAS levels off.
[17]This is similar to the results found in Robert Merton, “On the Pricing of Corporate Debt: The Risk Structure of interest Rates,” *Journal of Finance* (May 1974).

Exhibit 9: OAS versus Term to Maturity: Default Option
(at different LTVs)



Valuing the Prepayment Option

Thus far we have assumed that the commercial mortgage was completely non-callable. In reality, while most commercial mortgages are more call-protected than residential mortgages, many have free periods. For example, the borrower in a 10-year balloon mortgage may be locked-out from prepayment for five years, and then is permitted to prepay. In addition, many commercial mortgages do not have lock-out provisions, but instead prepayment penalties which come in a variety of forms. The investor in the mortgage needs an ability to place a value on the lock-out and/or penalties in order to properly compare investment alternatives.

In order to analyze the effects of callability we continue with our example where we assumed a commercial mortgage with a 10-year maturity, 7% coupon, initial DSCR of 1.30x, initial LTV of 70%, initial 10-year Treasury at 4.76%, interest rate volatility of 11%, and NOI volatility of 16%. We assume that the loan follows a 30-year amortization schedule and has a balloon payment at the end of year 10 (which implies a 9.4-year weighted-average life). In addition, we assume that the loan is fully callable after 5 years without penalty at par. (This is obviously very favorable for the borrower. In practice, there would likely be some sort of penalty structure following the lock-out period.)

Naturally, if the loan is also callable, the investment is less attractive. As a result, the OAS is lower as can be seen in Exhibits 10, 11, and 12. It is interesting to note that the difference gets larger for lower quality loans. This is because the option to call-in lower quality debt is even more valuable.

In addition to the issues of callability there are factors which, while not discussed here, have an impact on value and show up in the OAS. These include the particular penalty structure; what happens in a default, extension, and modification provision in a balloon default; the price of the loan (assumed in all of the above analysis to be priced at par[18]); the shape of the yield curve; etc. All of these can be analyzed by the model.

[18]If the investor purchased the loan at a deep discount, a default could be a favorable event if the proceeds from the foreclosure are sufficient. Similarly, a loan purchased at a premium will suffer if there is a default even if the foreclosure process only recovers the par amount.

Exhibit 10: OAS versus DSCR: Refinance and Default Options

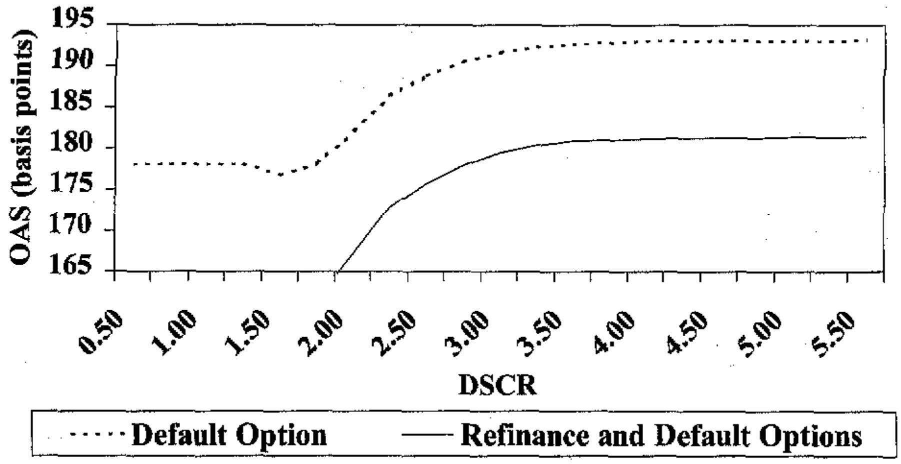


Exhibit 11: OAS versus LTV: Refinance and Default Options

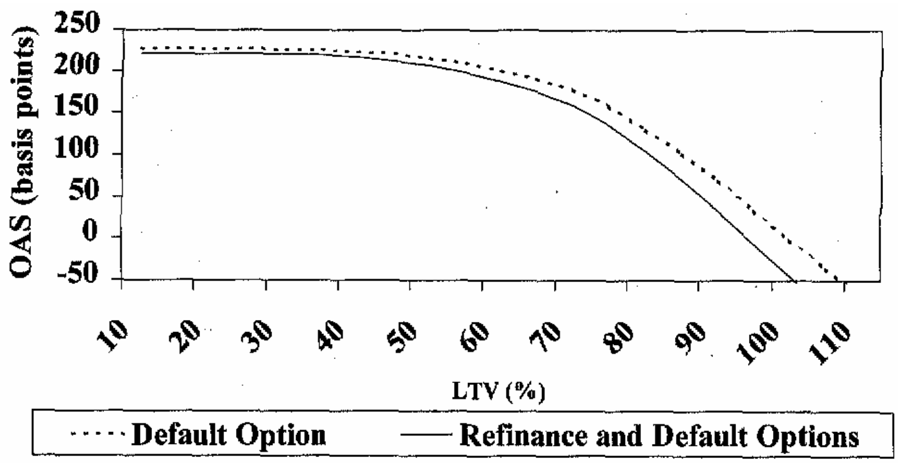
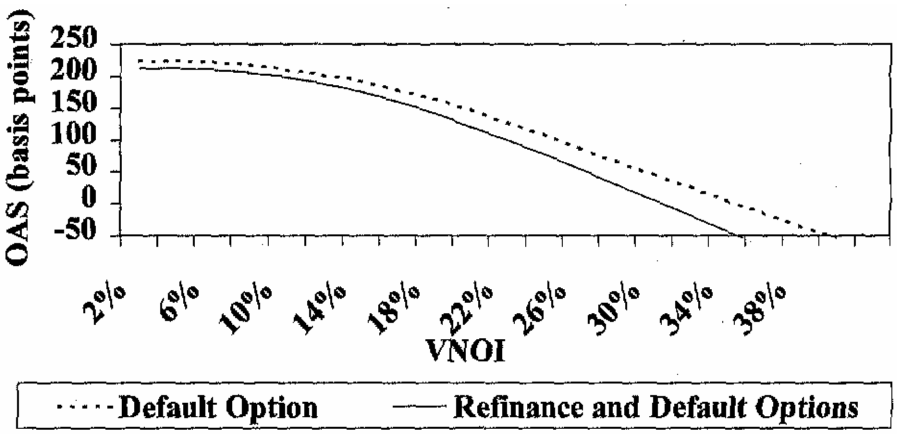


Exhibit 12: OAS versus Volatility of NOI: Refinance and Default Options



VALUING MULTI-CLASS COMMERCIAL MORTGAGE-BACKED SECURITIES[19]

Thus far, we have applied the model to the valuation of a commercial mortgage. In this section we use the model to analyze multi-class commercial mortgage-backed securities. Securitization is the process of converting financial assets that produce cash flows into securities that trade in the financial markets. The cash flows from the loans are used to pay the certificate holders. Often many loans are pooled together in the securitization. A single class of bondholders can be created or there can be numerous classes. In addition to creating liquidity, the securitization process can be used to alter credit quality through various means of credit enhancements. These credit enhancements can take the form of guarantees, over collateralization, or senior-subordination. Today, senior-subordination is used almost exclusively. By creating a credit and payment prioritization, credit risk is reallocated among the bond classes. Typically, all principal payments that are made by the borrower(s) are used to pay off the most senior outstanding bond class, whereas losses are allocated to the most junior outstanding class. This process effectively decreases the LTV of the most senior bond class.

Because the losses are mostly absorbed by the junior classes, the senior class has more credit protection than its underlying collateral. By utilizing subordination, one can create securities which have higher credit ratings than the underlying collateral. The rating agencies set the effective LTV and DSCR requirements in each deal in order to achieve the desired bond ratings.

A Simple Sequential CMBS Structure

The question which investors would like to be answered is: Which has better value — the AAA bond at +85 bp or the BBB bond at +190 bp? Our model can be used to help answer this type of question. We use the following example to demonstrate the application of the model.

We start with the same type of loan as in the prior section, i.e. a 7% coupon, 10-year balloon loan with a 30-year amortization schedule, 10-year Treasury rates at 4.76%, NOT volatility of 16%, interest rate volatility of 11%, and a correlation of 20% between interest rates and NOI. For analytic purposes we assume that instead of a single loan, we have a pool of ten $20 million loans with LTVs ranging from 61.6% to 81.1% (weighted average 70%) and DSCRs ranging from 1.48x to 1.12x (weighted average 1.30x).

[19]Note that there will be some differences in the valuation results since in this section we use Monte Carlo simulation rather than obtaining an exact solution from the binomial tree.

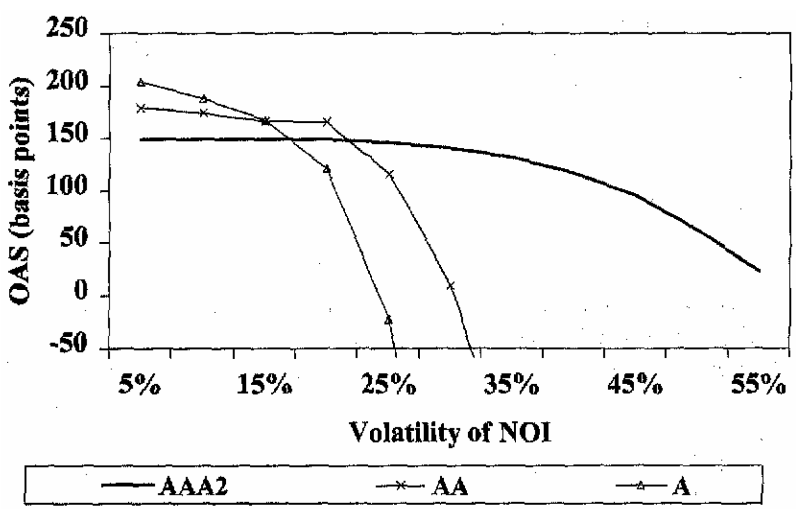
Exhibit 13 depicts a typical CMBS deal.[20] In the deal, the N/R (unrated) class is structured as the most junior tranche. There are two AAA classes, one short maturity and one long maturity. The other classes range from AA to B-. In addition there is an IO class stripped from the AAA through the A rated classes.

As was the case with the unstructured commercial mortgage, when the NOI volatility is low, the OAS is almost equal to the stated yield spread.[21] At higher levels of NOI volatility, the OAS declines. As expected, the largest declines take place in the most junior classes since they are the first to absorb losses caused by defaults. Interestingly, the OAS also declines substantially for the most senior class. This is because the recoveries from defaults are used to pay the most senior classes first. Since, in this example, the senior classes are priced at a premium, the premature receipt of principal at par leads to a degradation in the bond's yield. In Exhibits 14 and 15 we show the OAS versus NOI volatility for the senior bonds.

There are several other points worth noting. First, the most protected classes are the long AAA and the AA. Their OAS changes the least as a function of NOl volatility since they are not the first to absorb losses or recoveries. Second, the A rated CMBS appears to be a better value than the underlying loan if the two are offered at the same stated spread for a wide range of NOl volatilities. Finally, the lowest rated classes appear to be the cheapest for low levels of NOl volatility.

In Exhibit 16, we allow the loans to be callable after 5 years without penalty. This substantially hurts the value of the collateral and the long AAA. The greatest damage is to the IO class. This is why a complete prepayment lockout or at least a substantial yield maintenance allocation to these classes is so important and valuable. The subordinate classes can actually benefit as shown by the improvement in their OAS because they are priced at a discount.

Exhibit 14: OAS versus Volatility of NOI for Senior Tranches: Default Option



[20]In this example, the spreads are reasonably representative of the market as of the first quarter of 1999.
[21]Since the Treasury curve is positively sloped the OAS would still be below the stated yield spread due to the dispersion of the cash flows.

Exhibit 15: OAS versus Volatility of NOI for Senior Tranches: Refinance and Default Options

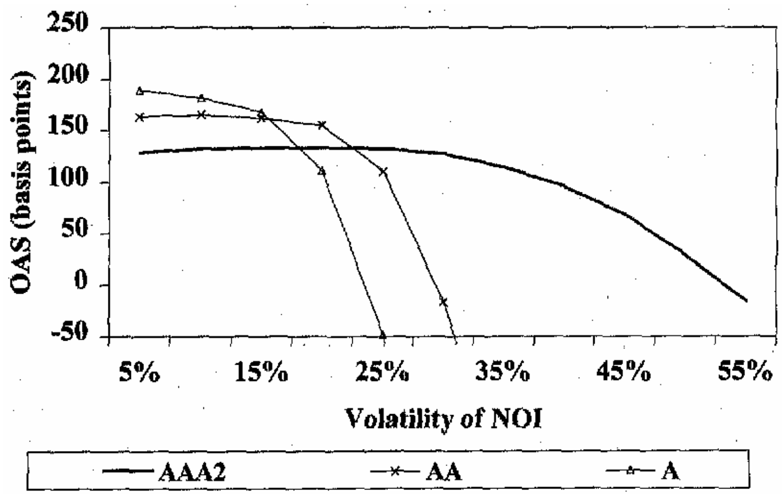


Exhibit 16: An Example of a Sequential Structure of CM8S: Refinance and Default Options (10 Loans)

Class	Size ($mm)	WAL	Price	Yield Spread	OAS				
					Vol. 6%	Vol. 9%	Vol. 16%	Vol. 20%	Vol. 30%
Collateral	200.0	9.40	100.00	226	202	185	111	58	−79
AAA Short	24.5	5.14	101.50	138	129	130	122	116	107
AAA Long	119.5	9.99	101.50	146	130	131	133	133	127
AA	12.0	10.00	100.00	176	164	166	159	155	−17
A	11.0	10.00	100.00	201	189	184	159	111	−322
BBB	9.0	10.00	96.05	281	273	257	160	16	−821
BBB−	3.0	10.00	89.60	381	377	346	157	−99	−1267
BB+	8.5	10.00	77.31	600	600	539	171	−310	−1637
BB	2.0	10.00	76.05	625	596	506	−139	−836	−2159
BB−	2.0	10.00	67.96	800	771	657	−124	−858	−2293
B+	3.5	10.00	63.86	900	840	656	−375	−1209	−2626
B	1.0	10.00	61.93	950	845	547	−779	−1886	−2681
B−	1.5	10.00	56.61	1100	971	551	−986	−1874	−2585
N/R	2.5	10.00	29.20	2424	2395	1738	−46	−650	−1374
IO	200.0	5.07	2.75	500	−248	−353	−515	−543	−471

Subordinate Bonds

The subordinate bond classes (B-pieces) in a CMBS deal are usually priced at a discount. Our analysis shows that the OAS of the B-pieces remains wide compared to the senior tranches when a volatility as high as 9% is assumed for the NOI. (See Exhibits 17 and 18.)

Interest-Only Strips

As we noted earlier, since senior bonds are priced with lower yields than the collateral, they would have prices substantially above par if they had the same coupon. Since many investors do not want to purchase premium bonds, interest is stripped off and interest-only (IO) classes are created.[22] Interest is usually stripped in order to price the senior class close to par or at low premium in the CMBS deal. It is in the interest of the issuer to create the strip class if the proceeds from the strip class and the par bond is greater than the proceeds from a premium bond. Since the price of an IO strip is more sensitive than its principal bond with the same credit rating, it is traded at a much wider spread.

Exhibit 17: OAS versus Volatility of NOI for Subordinate Tranches: Default Options

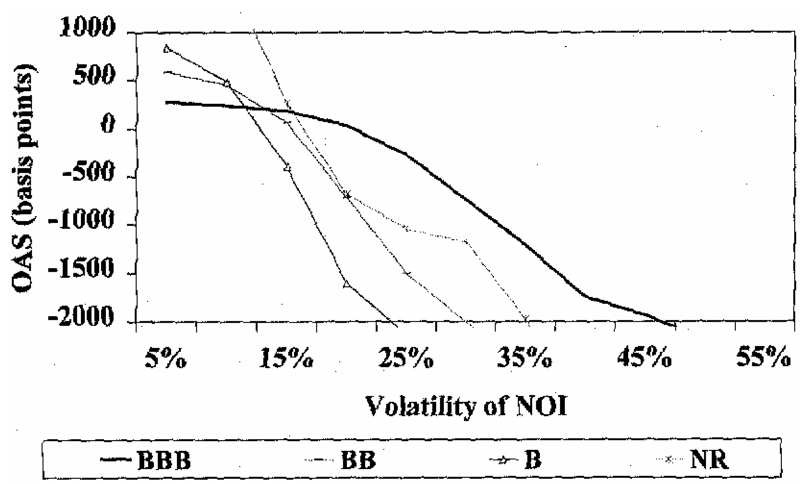
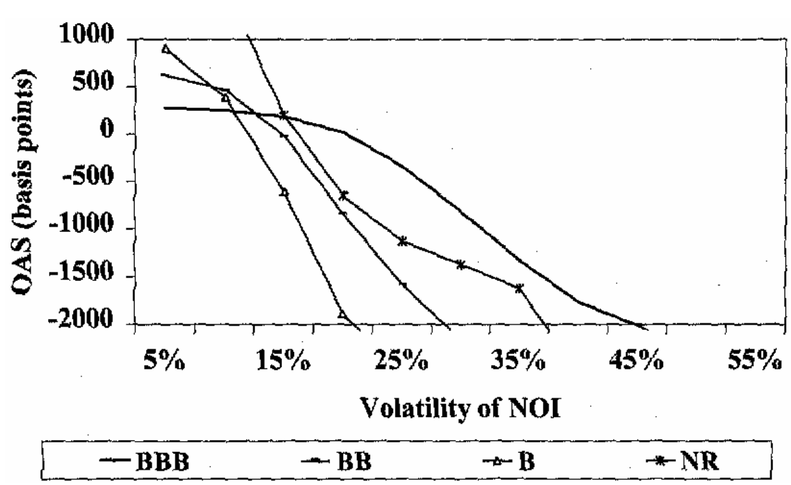


Exhibit 18: OAS versus Volatility of NOI for Subordinate Tranches: Refinance and Default Options



[22]This can be done either at the loan level or the bond class level.

IO strip classes have special characteristics, which make the use of an OAS model in their valuation extremely useful. Since interest is paid only when the notional principal is outstanding, when the notional principal upon which the IOs payments are based is reduced, the amount of interest paid to the IO holder declines. There are two ways in which the erosion of notional principal can occur. The reduction in principal takes place due to either losses or due to principal payments. In the case of a loss, the senior-subordinated structure requires that the loss be allocated to the most junior outstanding class. Thus, IOs which are stripped from the AAA bond are relatively insulated from erosion of principal due to the allocation of losses. On the other hand, erosion of principal also occurs due to principal payments. The senior-subordinated structure requires that the most senior outstanding class receive principal payments. Thus, even a minor amount of prepayments leads to the erosion of principal associated with the IO stripped from the AAA bond. Principal payments can be either voluntary or involuntary. Voluntary prepayments are related to refinancing or the selling of the property. The IO holder, thus, derives considerable protection from prepayment protection features.[23]

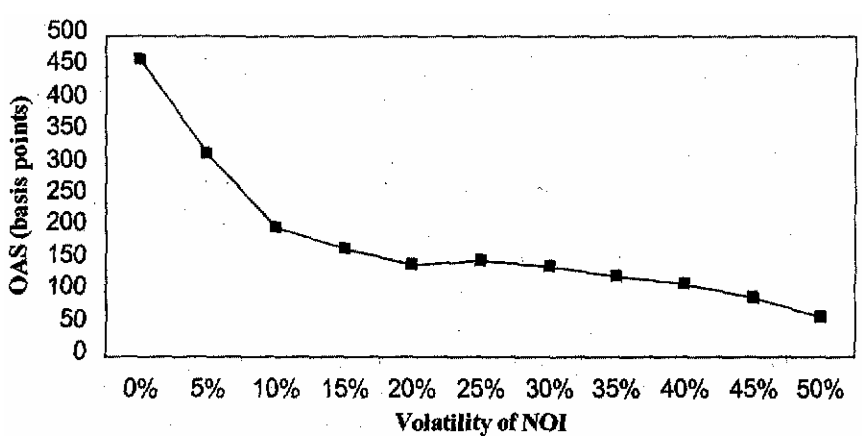
Involuntary principal payment is a slightly more subtle concept. It arises from recoveries from foreclosure proceedings. These recoveries also must go to the most senior outstanding class. Thus, high defaults with a high rate of recovery also can erode the principal of the IO stripped from the most senior bond. The senior IO holder is in the odd position in the case of a default of hoping for zero recovery.

Exhibit 19 shows that the OAS for the IO class remains high at relatively high levels of NOI volatility, whereas Exhibit 13 shows how poorly the IO performs when the loans are prepayable. Investors should be willing to pay up substantially for IO's that are backed by well call protected loans. Note that while we assumed immediate foreclosure, the foreclosure process on average takes about a year. This would substantially improve the value of the IO classes.

DISPERSION OF LOAN QUALITY

Investors need to give serious consideration to the diversity of the loans backing the bond classes in a CMBS deal. In the prior CMBS examples, we assumed there were ten loans with a range of LTVs and DSCRs. While the average LTV was 70% and the average DSCR was 1.3x, the dispersion in quality generally has a negative impact on the bond classes. In Exhibits 20 and 21 we compare the OAS versus NOl volatility for the AAA bond and the B rated class assuming a ten loan portfolio and a single loan portfolio. One can see that the dispersion lowers the OAS for the B rated class. As a junior class, it suffers the downside of the lower quality loans without an equal benefit from the higher quality loans. The AAA bond on the other hand gets the benefit of the diversity, because the lower quality loans default, and the remaining pool consists of much higher quality loans.

Exhibit 19: OAS versus Volatility of NOI for Interest-Only Strip: Default Option



[23]Lockout, however, provides better protection than yield maintenance for two reasons. First, while yield maintenance serves as a disincentive, it is possible that in the event of prepayment the IO holder may not be allocated his fair share. More importantly, as we mentioned earlier property owners may want to prepay even when interest rates rise. In this instance there may be no yield maintenance to distribute.

Exhibit 20: Effect of Loan Quality Dispersion: A/LA CMBS Bond Class

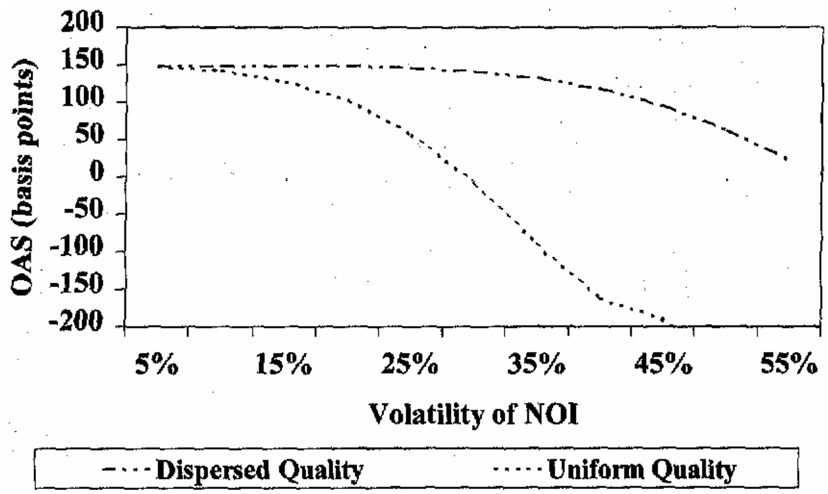
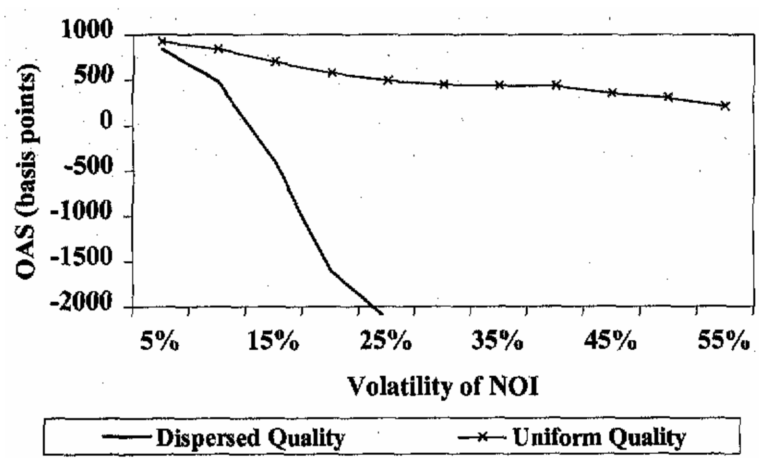


Exhibit 21: Effect of Loan Quality Dispersion: B CMBS Bond Class



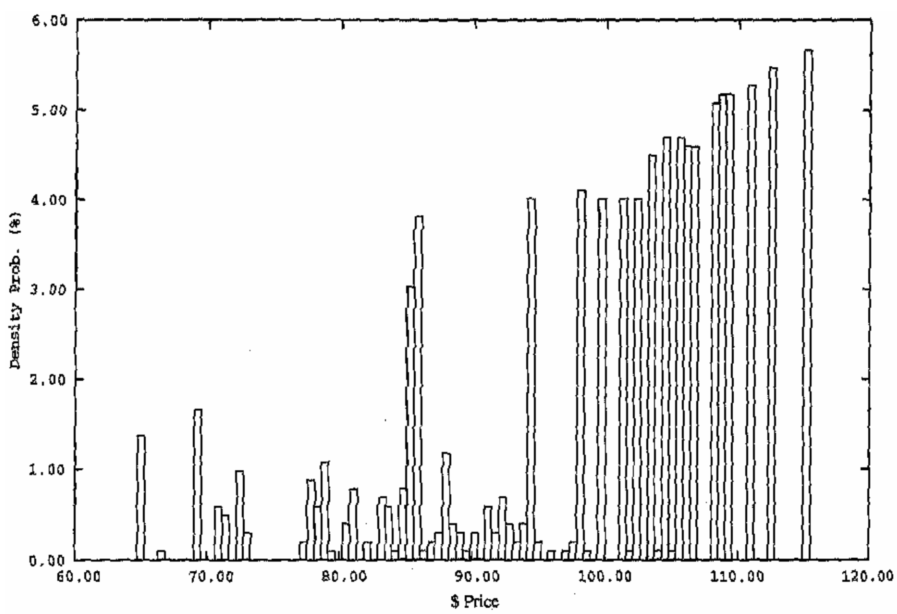
RELATIVE RISK OF CMBS BOND CLASSES

Up to this point we have used OAS model mostly as a tool for pricing and assessing relative value. The model, because it is based on the evaluation of the securities in the probability space, can also be used to determine relative risk. We use the prices that are obtained from the price distribution. One would expect that the price variation for the less risky securities be smaller (as a percent of the security's price) than for the more risky securities. This is generally the case. In Exhibits 22, 23, and 24 we show the price distribution resulting from the model assuming NOI volatility of 16%. To focus on the credit risk we held interest rates constant. Notice how much tighter the price range is for the AAA bond than for the collateral; look how wide the range is for the B rated class.

In Exhibit 25 we summarize the statistics of the distributions for each of the bond classes. In general, the lower rated classes have the highest risk as measured by the coefficient of variation.[24]

If we use the coefficient of variation as our risk measure, and OAS as our measure of relative value, we can construct the "efficient frontier" for investment analysis. Certain bonds lie on the boundary of the efficient frontier. This suggests that portfolios consisting of these bonds can be formed to outperform combinations of the other bonds. Perhaps investors should take the ratio of OAS to the coefficient of variation as the measure of relative value, much like the Sharpe ratio in stock analysis.[25]

Exhibit 22: Price Distribution of Loan



[24]The coefficient of variation normalizes the risk by looking at the ratio of standard deviation to the mean.
[25]William F. Sharpe, "Mutual Fund Performance," *Journal of Business* (January 1966).

Exhibit 23: Price Distribution of CMBS – AAA

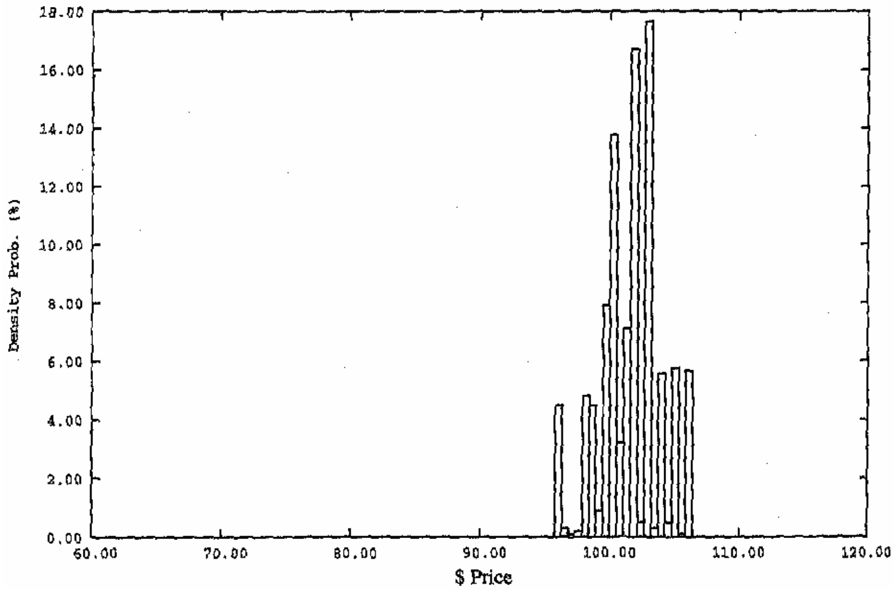


Exhibit 24: Price Distribution of CMBS – B- Class

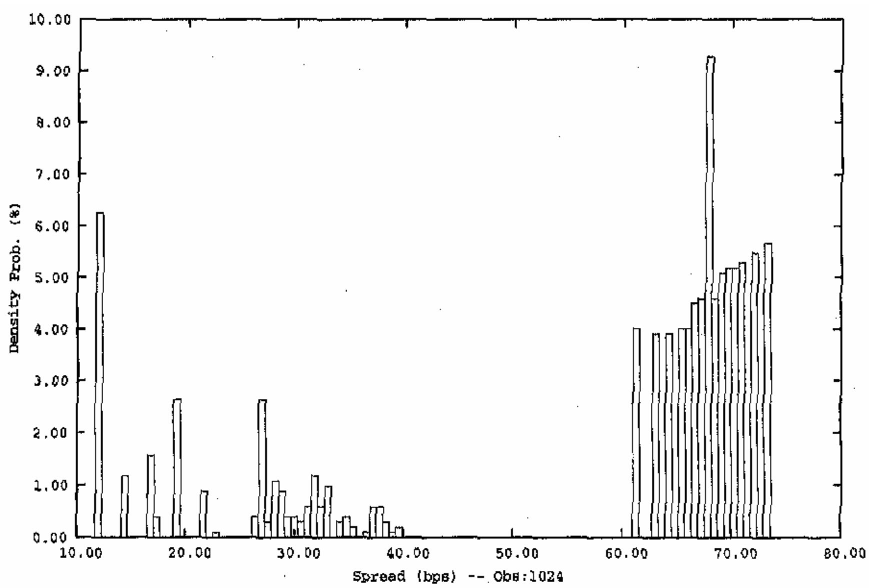


Exhibit 25: CMBS Summary Statistics

	Mean	Standard Deviation	OAS	Coefficient of Variation
Col	100.00	12.27	150.00	12.27
AAA1	101.50	2.36	134.46	2.32
AAA2	101.50	7.97	133.22	7.85
AA	100.00	26.67	49.34	26.67
A	100.00	33.49	−2.72	33.49
BBB1	96.05	39.86	3.53	41.50
BBB2	89.60	36.69	100.96	40.95
BB1	77.31	30.51	315.69	39.47
BB2	76.05	30.00	338.14	39.44
BB3	67.96	25.92	511.94	38.14
B1	63.86	23.87	611.32	37.38
B2	61.93	22.91	661.03	37.00
B3	56.61	20.29	810.24	35.85
NR	28.72	7.36	2,172.75	25.64
IO	2.75	0.70	218.81	25.57

Finally in Exhibit 26 we present the same information as in Exhibit 25 but instead of using one loan or (uniform quality) portfolio, we use a portfolio with the same average quality but with dispersion among the loans. One can clearly see how the diversity lowers the standard deviation of the AAA bond, but dramatically raises it for the lower rated classes.

Exhibit 26: CMBS Summary Statistics (Diverse Pool)

	Mean	Standard Deviation	OAS	Coefficient of Variation
Col	100.00	8.64	140.00	8.64
AAA1	101.50	1.51	116.65	1.49
AAA2	101.50	5.16	148.85	5.09
AA	100.00	9.99	165.04	9.99
A	100.00	15.40	159.77	15.40
BBB1	96.05	23.62	165.93	24.59
BBB2	89.60	30.49	170.44	34.03
BB1	77.31	33.52	204.60	43.36
BB2	76.05	49.31	−70.72	64.84
BB3	67.96	48.77	−60.19	71.76
B1	63.86	56.87	−294.44	89.07
B2	61.93	72.34	−617.27	116.81
B3	56.61	75.67	−771.63	133.68
NR	29.20	34.44	−17.95	117.95
IO	2.75	0.28	193.81	10.09

CONCLUSION

In this chapter we introduced a model which uses the borrower's behavior to construct a decision payoff matrix for exercising default and prepayment options and utilizes option theory to value these options. The model employs interest rates and the net operating income of the mortgaged property as the two underlying factors. Based upon the model, the option values can be assessed. We then derived an option-adjusted spread (OAS) to measure the relative value of commercial mortgages and commercial mortgage-backed securities. Given a market price, the OAS decreases as the risk exposure increases. The model serves as a unifying framework, bringing real estate valuation, fixed-income pricing, and option theory together. Factors affecting the real estate, mortgages, and structure are evaluated simultaneously across a wide range of scenarios. This analysis helps reveal hidden weaknesses in the structure and enables the analyst to compare deals.

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